MATH 118: Midterm 1 Key

Name: ______________________________

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) True or False: We are allowed to use exponent laws in the following way:

$$\left(\frac{x-2}{x^2y}\right)^2 = \frac{x^2-2^2}{x^4y^2}$$

   False. The x and 2 in the numerator are terms. Exponents = multiplication = **factors**. Properties for factors do not interact with terms.

   There is **only one property** where terms and factors interact. It is the distributive property.

   (b) True or false: We can simplify

$$\frac{\overbrace{(x-1)^2x}^{\text{Term}} - \overbrace{1}^{\text{Term}}}{(x-1)}$$

   by crossing out the $(x-1)$ (the one in parentheses).

   False. The numerator has global terms. Therefore you cannot have global factors. You can only cancel global factors.

   (c) Suppose

$$f = x^2 - x \qquad g = x(x-1)$$

   Expand and simplify $f - g$.

   Don't forget to distribute the negative to **both terms.**

$$\begin{aligned} f - g &= x^2 - x - x(x-1) \\ &= x^2 - x - x^2 + x \\ &= \boxed{0} \end{aligned}$$

2. Factor and simplify:

Factor means to convert **terms** into **factors**. Do not expand.

(a) $x^2 - 1$

Two term factoring problem. Can't use GCF.

Using $A^2 - B^2$ where $A = x$ and $B = 1$, we have

$$\overset{A^2}{x^2} - \overset{B^2}{1} = \boxed{(x-1)(x+1)}$$

(b) $x^3 - 3x^2 - x + 3$

Four term factoring problem. Can't use GCF.

Using grouping:

$$\begin{aligned} x^3 - 3x^2 - x + 3 &= x^2(x-3) - (x-3) \\ &= (x-3)(x^2-1) \\ &= \boxed{(x-3)(x-1)(x+1)} \end{aligned}$$

(c) $\underbrace{4(2x+1)(x-2)^2}_{\text{Term}} + \underbrace{2(x-2)(2x+1)^2}_{\text{Term}}$

Two term factoring problem. Can use GCF, common factor is $2(2x+1)(x-2)$. Undo distributive law:

$$\begin{aligned} 4(2x+1)(x-2)^2 + 2(x-2)(2x+1)^2 &= 2(2x+1)(x-2)\left[2(x-2) + (2x+1)\right] \\ &= 2(2x+1)(x-2)(2x-4+2x+1) \\ &= \boxed{2(2x+1)(x-2)(4x-3)} \end{aligned}$$

3. Expand and simplify:

(a) $(x+h)-1-(x-1)$

Note that $(x+h)-1=(x+h)+(-1)$. Do not distribute negatives backwards; the -1 is a different term.

$$\begin{aligned}(x+h)-1-(x-1) &= x+h-1-x+1\\ &= \boxed{h}\end{aligned}$$

(b) $3(2x+1)-2(x+1)^2x$

I will expand $(x+1)^2=x^2+2x+1$ first. Remember the parentheses.

$$\begin{aligned}3(2x+1)-2(x+1)^2x &= 6x+3-2(x^2+2x+1)x\\ &= 6x+3-2x^3-4x^2-2x\\ &= \boxed{-2x^3-4x^2+4x+3}\end{aligned}$$

4. Simplify:

(a) $\dfrac{x}{2x-1} - \dfrac{x}{x-1}$

Left fraction missing $(x-1)$ as a factor, right missing $(2x-1)$ as a factor. Introduce and don't forget parentheses due to multiplying 2 terms:

$$\begin{aligned}
\frac{x}{2x-1} - \frac{x}{x-1} &= \frac{(x-1)}{(x-1)} \cdot \frac{x}{2x-1} - \frac{x}{x-1} \cdot \frac{(2x-1)}{(2x-1)} \\
&= \frac{(x-1)x}{(x-1)(2x-1)} - \frac{x(2x-1)}{(x-1)(2x-1)} \\
&= \frac{(x-1)x - x(2x-1)}{(x-1)(2x-1)} \\
&= \frac{x^2 - x - 2x^2 + x}{(x-1)(2x-1)} \\
&= \frac{-x^2}{(x-1)(2x-1)} \\
&= \boxed{-\frac{x^2}{(x-1)(2x-1)}}
\end{aligned}$$

(b) $\dfrac{2(x+h)^2 - 3 - (2x^2 - 3)}{h}$

Expand numerator because like terms are created.

$$\begin{aligned}
\frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h} &= \frac{2(x^2 + 2xh + h^2) - 3 - 2x^2 + 3}{h} \\
&= \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\
&= \frac{4xh + 2h^2}{h} \\
&= \frac{\cancel{h}(4x + 2h)}{\cancel{h}} \\
&= 4x + 2h \\
&= \boxed{2(2x+h)}
\end{aligned}$$

(c) $\dfrac{\dfrac{1}{x-1}-\dfrac{1}{x+1}}{x+1}$

Compound fraction. Imagine the numerator is a separate problem.

Left missing $(x+1)$, right missing $(x-1)$.

$$\begin{aligned}
\frac{\dfrac{1}{x-1}-\dfrac{1}{x+1}}{x+1} &= \frac{\dfrac{x+1}{x+1}\cdot\dfrac{1}{x-1}-\dfrac{1}{x+1}\cdot\dfrac{x-1}{x-1}}{x+1} \\
&= \frac{\dfrac{x+1}{(x+1)(x-1)}-\dfrac{x-1}{(x+1)(x-1)}}{x+1} \\
&= \frac{\dfrac{x+1-(x-1)}{(x+1)(x-1)}}{x+1} \\
&= \frac{\dfrac{x-1-x+1}{(x+1)(x-1)}}{x+1} \\
&= \frac{\dfrac{2}{(x+1)(x-1)}}{x+1} \\
&= \frac{2}{(x+1)(x-1)}\cdot\frac{1}{x+1} \\
&= \boxed{\frac{2}{(x+1)^2(x-1)}}
\end{aligned}$$

5. Perform the given instruction:

   (a) Simplify $-8^{\frac{2}{3}}$

   Memorize the Compendium. Follow the definitions of negative and fractional exponent:

$$\begin{aligned}
-8^{\frac{2}{3}} &= (-1)\cdot 8^{\frac{2}{3}} \\
&= (-1)\cdot \sqrt[3]{8^2} \\
&= (-1)\cdot \sqrt[3]{(2^3)^2} \\
&= (-1)\cdot \sqrt{32^6} \\
&= (-1)\cdot 2^{\frac{6}{3}} \\
&= (-1)\cdot 2^2 \\
&= \boxed{-4}
\end{aligned}$$

   (b) Solve the equation

$$8x^2 - 10x = 7$$

   Presence of x^2 is a **quadratic equation** $ax^2 + bx + c = 0$:

$$\begin{aligned}
8x^2 - 10x &= 7 \\
8x^2 - 10x - 7 &= 0
\end{aligned}$$

   $a = 8, b = -10$ and $c = -7$.

   One diagonal product $2 \cdot -7 = -14$ which is close to b so try it

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 4     7
 |  X  |
-2     1
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   which works, cross-product and sum is $-14 + 4 = -10 = b$. So:

$$\begin{aligned}
8x^2 - 10x &= 7 \\
(4x-7)(2x+1) &= 0
\end{aligned}$$

$$4x - 7 = 0 \qquad 2x + 1 = 0$$

$$4x = 7 \qquad 2x = -1$$

$$\boxed{x = \frac{7}{4}} \qquad \boxed{x = -\frac{1}{2}}$$

(c) Isolate the variable y:

$$3xy - 2(xy + y) - x^2 = y$$

Follow the four steps.

$$3xy - 2(xy + y) - x^2 = y$$

$$3xy - 2xy - 2y - x^2 = y$$ Step 1: Expand everything

$$3xy - 2xy - 2y - y = x^2$$ Step 2: Terms with y on one side

$$xy - 3y = x^2$$ Combine like terms

$$y(x - 3) = x^2$$ Step 3: Factor out y

$$y = \frac{x^2}{x - 3}$$ Step 4: Divide by factor attached to y

Solution: $\boxed{y = \frac{x^2}{x - 3}}$